Mathematics: NUMBERS & OPERATIONS

Types of Numbers

Natural: non-zero positive number with no decimal
▶ 1, 2, 3, ...

Whole: positive number with no decimal
▶ 0, 1, 2, 3, ...

Integer: number with no decimal
▶ ... , –2, –1, 0, 1, 2, ...

Rational: number that can be written as a fraction
▶ 2/3, –6, 0.725

Real: number that can be placed on a number line
▶ 2/3, –6, 0.725, π

Imaginary: number that includes i, where $i = \sqrt{-1}$
▶ $3i$, $6 + 2i$

Properties of Numbers

Commutative property: order doesn't matter
▶ $(-2)(3) = (3)(-2)$

Associative property: parts can be regrouped without changing the result
▶ $-3 + (-5 + 4) = (-3 + -5) + 4$

Distributive property: a product of sums can be written as a sum of products
▶ $a(b + c) = ab + ac$

Identity property: an operation on a produces a
▶ 1 ($a \times 1 = a$); 0 ($a + 0 = a$)

Order of Operations

P ▶ expressions inside parentheses, brackets and braces
E ▶ exponents and square roots
MD ▶ multiplication and division in order from left to right
AS ▶ addition and subtraction in order from left to right

Exponents

$a^0 = 1$

$a^n = \frac{1}{a^n}$

$a^m a^n = a^{m+n}$

$(a^m)^n$

$\frac{a^m}{a^n} = a^{m-n}$

$(ab)^n = a^n b^n$

$\frac{a}{b}^n = \frac{a^n}{b^n}$

Units of Measurement

Dimension	American	SI
length	inch/foot/yard/mile	meter
mass	ounce/pound/ton	gram
volume	cup/pint/quart/gallon	liter
force	pound-force	newton
pressure	pound-force per square inch	pascal
work and energy	cal/British thermal unit	joule
temperature	Fahrenheit	kelvin

Percentages

▶ part = whole × percent

▶ percent = $\frac{\text{part}}{\text{whole}}$

▶ whole = $\frac{\text{part}}{\text{percent}}$

Radicals

$\sqrt[b]{ac} = \sqrt[b]{a}\sqrt[b]{c}$

$\sqrt[b]{\frac{a}{c}} = \frac{\sqrt[b]{a}}{\sqrt[b]{c}}$

$\sqrt[b]{a^c} = a^{\frac{c}{b}}$

Percent Change

▶ amount of change = original amount × percent change

▶ percent change = $\frac{\text{amount of change}}{\text{original amount}}$

▶ original amount = $\frac{\text{amount of change}}{\text{percent change}}$

Prefixes

tera	10^{12}
giga	10^{9}
mega	10^{6}
kilo	10^{3}
hecto	10^{2}
deca	10^{1}
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deci	10^{-1}
centi	10^{-2}
milli	10^{-3}
micro	10^{-6}
nano	10^{-9}
pico	10^{-12}

Conversion Factors

1 in. = 2.54 cm	1 lb. = 0.454 kg
1 yd. = 0.914 m	1 cal = 4.19 J
1 mi. = 1.61 km	$1^{\circ}F = \frac{5}{9}(^{\circ}F - 32^{\circ}C)$
1 gal. = 3.785 L	1 cm^3 = 1 mL
1 oz. = 28.35 g	1 hr = 3600 s

Fractions

$\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$

$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

$\frac{a}{b} \div \frac{c}{d} = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right) = \frac{ad}{bc}$

Proportions

$\frac{a}{b} = \frac{c}{d} \rightarrow ad = bc$

Sequences and Series

Arithmetic	Geometric
$a_n = a_1 + d(n-1)$	$a_n = a_1 \times r^{n-1}$
$a_n = a_m + d(n-m)$	$a_n = a_m \times r^{n-m}$
$S_n = \frac{n(a_1 + a_n)}{2}$	$S_n = \frac{a_1(1-r^n)}{1-r}$
	$S_\infty = \frac{a}{1-r}(\lvert r\rvert < 1)$

d = common difference
a_n = nth term
n = number of the term
a_m = mth term
m = number of the term
a_1 = first term
S_n = sum through the nth term
r = the common ratio
S_∞ = sum of all terms